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Solutions to written exam for the M. Sc in Economics
Economics of Exchange Rates

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1. This question relates to the following learning objectives. Knowledge: Describe how the foreign exchange market is organized and how trades take place in the market; and describe and explain Covered Interest Rate Parity (CIP), Uncovered Interest Rate Parity (UIP), and Purchasing Power Parity (PPP) and be able to summarize the empirical evidence on international interest these parity conditions. Skills: Describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model, the portfolio balance model and Lucas asset pricing model) and apply these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models; describe the channels by which central bank intervention can affect the exchange rate and summarize the empirical evidence on these channels. Competences: Process relevant information for the analysis of the foreign exchange market; carry out economic analysis related to exchange rate determination, forecasting and international financial management.

   (a) In general, the central bank buys or sells foreign exchange in an attempt to influence future currency movements but we need to distinguish between

      - *Non–sterilized official intervention:* the monetary authorities buy or sell foreign exchange such that the monetary base is affected.
      - *Sterilized official intervention:* the monetary authorities buy or sell foreign exchange and sterilize the effects on the monetary base by simultaneously selling or buying domestic bonds.

      There are, in principle, two ways through which the central bank may influence the exchange rate: The portfolio balance channel and the signaling channel.

      The portfolio channel: Both non-sterilized and sterilized interventions. The former case refers to buying or selling foreign bonds (or assets) where the money stock is affected. In the latter case there is a swap of foreign bonds for domestic bonds (or the opposite) in the portfolios held by the central bank and the households while the money stock unchanged. Note that there will be no effect on the exchange rate if domestic and foreign bonds are perfect substitutes.

The signaling channel: Official intervention serving as a signal of future monetary policy by providing the foreign exchange market with new relevant information. A basin underlying assumption is that the current exchange rate is a function of current and discounted expected future fundamentals. An intervention, in this case, sends a signal to the market participants about future fundamentals. If future fundamentals are expected to change, the current exchange rate will also change. Even sterilized interventions affect expectations about future movements in the fundamentals with a feedback effect on the exchange rate. This effect also occurs in the monetary models where foreign and domestic bonds are perfect substitutes.

(b) It is stated in the problem that the expected change in the exchange rate is given by uncovered interest rate parity with a risk premium, i.e.,

$$E_t\left[\Delta s_{t+1} \mid \Omega_t\right] = i_t - i_t^* + \rho$$

and we know that

$$s_t = (1-\theta)\sum_{i=0}^{\infty}\theta^i E_t\left[f_{t+i} \mid \Omega_t\right]$$

A change in the exchange rate can be decomposed into two parts, the expected change given above and the unexpected change:

$$\Delta s_{t+1} = s_{t+1} - s_t = \underbrace{s_{t+1} - E_t\left[s_{t+1} \mid \Omega_t\right]}_{\text{unexpected change}} + \overbrace{E_t\left[s_{t+1} \mid \Omega_t\right] - s_t}^{\text{expected change}}$$

The expected change is given so it suffices to compute the unexpected change. Using the exchange rate equation above (for $t+1$) we have that

$$s_{t+1} = (1-\theta)\sum_{i=0}^{\infty}\theta^i E_{t+1}\left[f_{t+1+i} \mid \Omega_{t+1}\right]$$

Take the expectation of this using information available at t

$$E_t\left[s_{t+1} \mid \Omega_t\right] = E_t\left[(1-\theta)\sum_{i=0}^{\infty}\theta^i E_{t+1}\left[f_{t+1+i} \mid \Omega_{t+1}\right] \mid \Omega_t\right]$$

and subtract from the actual change to obtain

$$E_t\left[s_{t+1} \mid \Omega_t\right] = (1-\theta)\sum_{i=0}^{\infty}\theta^i\left(E_{t+1}\left[f_{t+1+i} \mid \Omega_{t+1}\right] - E_t\left[f_{t+1+i} \mid \Omega_t\right]\right)$$

and putting the two parts together we find

$$\Delta s_{t+1} = \left(i_t - i_t^* + \rho\right) + (1-\theta)\sum_{i=0}^{\infty}\theta^i\left(E_{t+1}\left[f_{t+1+i} \mid \Omega_{t+1}\right] - E_t\left[f_{t+1+i} \mid \Omega_t\right]\right).$$

Note that the unexpected change represents new information arriving after period t but before period $t + 1$.

There are two channels as explained above. Portfolio balance channel: actual interventions between period t and $t + 1$ affect the demand for domestic and foreign bonds and since it is assumed that these bonds are not perfect substitutes there will be a change in the risk premium ρ. Sterilized interventions will have an effect because of this. Signaling channel: both oral interventions and actual interventions influence the spot rate by directly altering expectations about future fundamentals, the unexpected change.

The main idea in this model is that the intervention is common knowledge and therefore observed by all agents. The intervention represents new information that affects the expectations of all agents leading them to revise their expectation about future fundamentals.

There is also a third possible channel, the coordination channel. This idea is based on the portfolio shift model where we distinguish between common knowledge and private information. In the present setting, market maker's determine the spot rate using common knowledge (or public information). When market maker's trade with their customers they also learn about their private information and this information is transmitted to all market maker's during their trading on the interbank market. This implies that not only the common knowledge (the actual intervention) but also private information, through the trading process, will affect the exchange rate. In this case, both portfolio adjustments and revisions of expected future fundamentals affect the exchange rate.

(c) The monetary policy is aimed solely at ensuring that the krone remains stable vis-a'-vis the euro. The government conducts its fiscal policy and economic policy in general so as to achieve stable economic development and in accordance with a fixed exchange rate. Since the main objective of the monetary policy in the euro area is to maintain price stability, low inflation is also created in Denmark. The central bank influences the krone rate through interventions and/or changes in monetary-policy interest rates. All interventions during normal times are sterilized but in turbulent markets, interventions are coordinated with changes in interest rates.

(d) Effectiveness of interventions: Interventions affect exchange rates in the short–run. Larger effects when intervention is combined with interest rate changes. Usually these results are viewed as consistent with the assumption that interventions signal monetary policy. Both international and Danish studies support this conclusion. Interventions are also more effective if they deviate from the prevalent policy. A statement or intervention that deviates from the current policy may have larger effects.

Oral interventions: This is found to be an effective policy tool for G3 policy-makers in influencing exchange rates. The effects of oral interventions are relatively large and effective when they go against the existing policy. Empirical results suggest that official communication has been an effective policy tool independent from the presence of actual interventions or particular monetary policy conditions. The effectiveness of both oral and actual interventions is related to the degree of market uncertainty and the positioning of participants in FX markets, suggesting that the effectiveness of interventions may primarily be related to the coordination channel. The results also provide support for micro-based approaches to exchange rate modeling and are consistent with the argument that oral and actual interventions function primarily through a coordination channel rather than a signaling channel.

Portfolio channel: Tests of the portfolio balance approach to official intervention are usually based on tests of the portfolio balance model or tests of the assumption of perfect substitutability. Empirical results suggest that there is only weak evidence supporting the portfolio balance channel.

Signaling channel: There are more empirical evidence supporting the signaling approach (both actual and oral interventions) than the portfolio balance approach.

2. This question relates to the following learning objectives. Knowledge: Describe how the foreign exchange market is organized and how trades take place in the market. Skills: Describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model, the portfolio balance model and Lucas asset pricing model) and apply these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models. Competences: Process relevant information for the analysis of the foreign exchange market; carry out economic analysis related to exchange rate determination, forecasting and international financial management.

(a) Basic assumptions are: Three assets (domestic and foreign bonds and money); free capital mobility, small open economy, prices and output fixed in the short-run; domestic and foreign bonds are not perfect substitutes. The main difference compared to standard monetary models is that domestic and foreign bonds are not perfect substitutes.

(b) To derive the four relationships we start with the total wealth equation (the first equation). Total wealth is given by

$$W \equiv M + B_p + SF_p$$

take total differential

$$dW = dM + dB_p + \overbrace{F_p dS + S dF_p}^{\text{use product rule}}$$

Next, we take the total differential of the wealth identity with respect to financial wealth W:

$$\mathrm{d}W - \frac{\partial M}{\partial W}\mathrm{d}W - \frac{\partial B}{\partial W}\mathrm{d}W - \frac{\partial F}{\partial W}\mathrm{d}W = 0$$

which implies that

$$\frac{\partial M}{\partial W}+\frac{\partial B}{\partial W}+\frac{\partial F}{\partial W}=1.$$

or

$$m_w+b_w+f_w=1 \tag{1}$$

An increase in wealth can be held as either money, domestic bonds or foreign bonds (the balance sheet constraint). The remaining two expressions are derived by taking the total differential of the wealth equation with respect to r and $E\dot{s}$. We then obtain

$$\overbrace{\frac{\partial M}{\partial r}}^{<0}+\overbrace{\frac{\partial B}{\partial r}}^{>0}+\overbrace{\frac{\partial F}{\partial r}}^{<0}=0$$

and

$$\overbrace{\frac{\partial M}{\partial E\dot{s}}}^{<0}+\overbrace{\frac{\partial B}{\partial E\dot{s}}}^{<0}+\overbrace{\frac{\partial F}{\partial E\dot{s}}}^{>0}=0.$$

These relations show that a higher interest rate will depress money demand and the demand for foreign bonds and increase the demand for domestic bonds. The reason is that a higher interest rate implies a lower price on domestic bonds, thus increasing demand. The second relationship implies that a higher expected exchange rate (a depreciation) reduces demand for domestic assets and raisning the demand for foreign assets. The sum of these changes in demand must balance.

(c) To derive the asset demand curves we make use of the demand functions (2) to (4). We start with the money market schedule. Take the total differential of the money demand function (2) with respect to r, W, $E\dot{s}$ and Y. We then obtain

$$dM=m_r dr+m_w dW+m_{E\dot{s}}dE\dot{s}+m_y dY$$

In equilibrium we know that $E\dot{s}=0$, $dM=0$ and we have assumed that $dY=0$ in the short-run. Therefore we obtain

$$0=m_r dr+m_w dW$$

From the total differential of the wealth equation above, we have that

$$dW=F_p dS$$

in equilibrium implying that

$$0=m_r dr+m_w F_p dS$$

such that the slope of the money market curve is

$$\frac{dr}{dS}=-\frac{m_w}{m_r}F_p>0$$

since $m_w > 0$ and $m_r < 0$.

The bond market schedule is derived by taking the total differential of the demand for domestic bonds and by imposing the equilibrium conditions we obtain

$$dB = b_r dr + b_w dW = b_r dr + b_w F_p dS$$

implying that

$$\frac{dr}{dS} = -\frac{b_w}{b_r} F_p < 0$$

since $b_w > 0$ and $b_r > 0$.

Finally, we derive the foreign bonds schedule. Take the total differential of the demand for foreign bonds (and imposing equilibrium conditions)

$$\overbrace{\frac{dSF_p}{dS} + \frac{dSF_p}{dF_p}}^{\text{use product rule}} = F_p dS = f_r dr + f_w dW$$

implying that

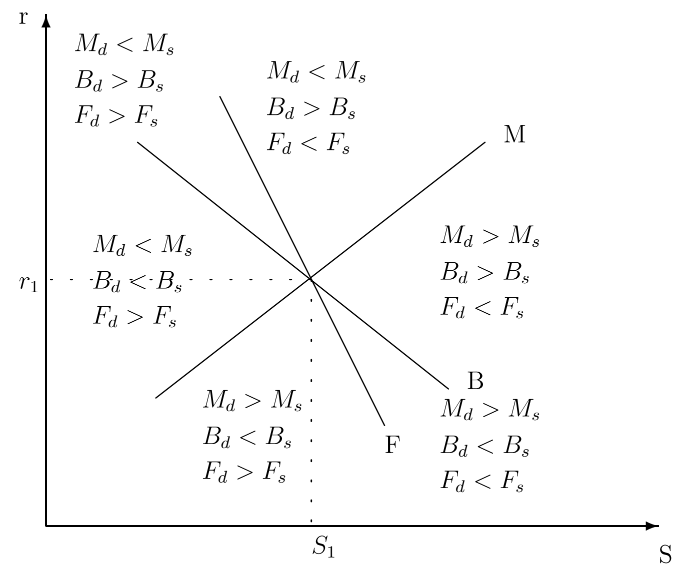
$$F_p dS = f_r dr + f_w F_p dS$$

which simplifies to

$$\frac{dr}{dS} = \frac{(1 - f_w)}{f_r} F_p < 0$$

since $f_r < 0$ and $0 < f_w < 1$.

(d) The model is illustrated below.



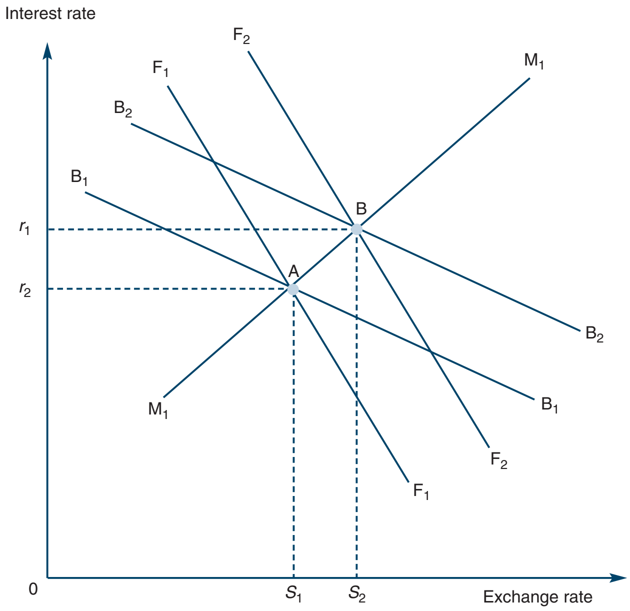
Intuition: The MM–schedule is upward sloping and describes equilibrium in the domestic money market. The explanation is that a depreciation of the exchange rate (an increase in S) leads to an increase in the domestic investor's wealth (foreign assets are worth more after the depreciation). The increase in wealth leads to an increase in the demand for money. But since the money supply is fixed, the increase in the money demand can only be offset by an increase in the interest rate.

The BB–schedule is downward sloping since a depreciation that raises wealth leads to an increase in the demand for domestic bonds, bond price will increase and the interest rate must fall. A depreciation must then be offset by a fall in the interest rate.

The FF–schedule depicting equilibrium on the market for foreign bonds is also downward sloping. The reason for this is that an increase in the interest rate leads to an increased demand for domestic bonds and therefore investors are inclined to sell money and foreign bonds to buy domestic bonds.

(e) We have a case when the exchange rate is strong implying increased demand for domestic assets. To defend the exchange rate, the central bank must provide domestic assets and buy foreign assets. In the first step, the central bank buys foreign bonds from the households providing domestic money. In the second step, the central bank sells domestic bonds to the households buying back the money. The total effect is a change in the currency positions of households portfolios from foreign bonds to domestic bonds leaving the money supply unchanged.

The effects are shown on the graph below. The excess supply of domestic bonds causes a shift in the BB schedule up to the right to B_2. There is excess demand for foreign bonds (the monetary authority buys foreign bonds) which will lead to a shift up to the right from F_1 to F_2. The economy moves from the initial equilibrium at point A to the new short-run equilibrium at point B. The total effect is an increase in the interest rate from r_2 to r_1 and an exchange rate depreciation from S_1 to S_2. These movements imply that the pressure on the krona fell, exactly what the policy intended to accomplish.



(f) We have derived the following equations

$$dB_p = b_r dr + b_w dW$$

$$dM = m_r dr + m_w dW$$

$$SdF_p = f_r dr + f_w dW - F_p dS$$

Rewrite these equations in matrix form

$$\begin{bmatrix} dB_p \\ SdF_p \\ dM \end{bmatrix} = \begin{bmatrix} b_r & b_w & 0 \\ f_r & f_w & -F_p \\ m_r & m_w & 0 \end{bmatrix} \begin{bmatrix} dr \\ dW \\ dS \end{bmatrix}$$

Solve this equation by pre-multiplying both sides with

$$\begin{bmatrix} b_r & b_w & 0 \\ f_r & f_w & -F_p \\ m_r & m_w & 0 \end{bmatrix}^{-1}$$

and using the hint we find that this inverse can be written as $\begin{bmatrix} \frac{m_w}{b_r m_w - b_w m_r} & 0 & -\frac{b_w}{b_r m_w - b_w m_r} \\ -\frac{m_r}{b_r m_w - b_w m_r} & 0 & \frac{b_r}{b_r m_w - b_w m_r} \\ \frac{f_r m_w - f_w m_r}{F_p(b_r m_w - b_w m_r)} & -\frac{1}{F_p} & \frac{b_r f_w - b_w f_r}{F_p(b_r m_w - b_w m_r)} \end{bmatrix}$

implying that

$$\begin{bmatrix} b_r & b_w & 0 \\ f_r & f_w & -F_p \\ m_r & m_w & 0 \end{bmatrix}^{-1} \begin{bmatrix} dB_p \\ SdF_p \\ dM \end{bmatrix} = \begin{bmatrix} b_r & b_w & 0 \\ f_r & f_w & -F_p \\ m_r & m_w & 0 \end{bmatrix}^{-1} \begin{bmatrix} b_r & b_w & 0 \\ f_r & f_w & -F_p \\ m_r & m_w & 0 \end{bmatrix} \begin{bmatrix} dr \\ dW \\ dS \end{bmatrix}$$

can be written as

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \begin{bmatrix} \frac{m_w}{b_r m_w - b_w m_r} & 0 & -\frac{b_w}{b_r m_w - b_w m_r} \\ -\frac{m_r}{b_r m_w - b_w m_r} & 0 & \frac{b_r}{b_r m_w - b_w m_r} \\ \frac{f_r m_w - f_w m_r}{F_p(b_r m_w - b_w m_r)} & -\frac{1}{F_p} & \frac{b_r f_w - b_w f_r}{F_p(b_r m_w - b_w m_r)} \end{bmatrix} \begin{bmatrix} dB_p \\ SdF_p \\ dM \end{bmatrix}$$

A sterilized foreign exchange operation discussed above implies that $dF_p = -dM$ and that $-dB_p = dM$ such that $dF_p = -dB_p$. Insert this

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \begin{bmatrix} \frac{m_w}{b_r m_w - b_w m_r} & 0 & -\frac{b_w}{b_r m_w - b_w m_r} \\ -\frac{m_r}{b_r m_w - b_w m_r} & 0 & \frac{b_r}{b_r m_w - b_w m_r} \\ \frac{f_r m_w - f_w m_r}{F_p(b_r m_w - b_w m_r)} & -\frac{1}{F_p} & \frac{b_r f_w - b_w f_r}{F_p(b_r m_w - b_w m_r)} \end{bmatrix} \begin{bmatrix} dB_p \\ -dB_p \\ 0 \end{bmatrix}$$

and simplify

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \begin{bmatrix} \frac{m_w}{b_r m_w - b_w m_r} dB_p \\ -\frac{m_r}{b_r m_w - b_w m_r} dB_p \\ -\frac{f_r m_w - f_w m_r + b_r m_w - b_w m_r}{F_p(b_r m_w - b_w m_r)} dB_p \end{bmatrix}$$

where we find that

$$dr = -\frac{m_w}{b_r m_w - b_w m_r} dB_p > 0$$

and

$$dS = -\frac{f_r m_w - f_w m_r + b_r m_w - b_w m_r}{F_p \left(b_r m_w - b_w m_r\right)} dB_p > 0$$

and

$$dW = \frac{m_r}{b_r m_w - b_w m_r} dB_p > 0$$

under our maintained assumptions. Use that $m_w = 1 - f_w - b_w$ implying that $b_r m_w - b_w m_r$ simplifies to $b_r(1 - f_w) > 0$ since $b_r > 0$ and $1 - f_w > 0$. We also note that $f_r m_w - f_w m_r + b_r m_w - b_w m_r = m_w(f_r + b_r) - m_r(f_w + b_w) > 0$ since $m_r < 0$ and $b_r + f_r = -m_r$.

The total short-run effects are identical to those obtained using the graphical analysis in the previous question.